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Name : ....

Reg. No: ....

COMBINED FIRST AND SECOND SEMESTER B.TECH (ENGINEERING) DEGREE EXAMINATION, APRIL 2013 (2K Scheme)

EN 2K 101 – MATHEMATICS – I
(Common to All Branches)

Time : Three Hours **Maximum : 100 Marks**

Answer all questions.

I (a) Evaluate $\lim_{x\to 0} \dfrac{2\cos x - 2 + x^2}{x^4}$

(b) Expand $\dfrac{\sin x}{x - \pi/4}$ about $x = \pi/4$.

(c) The torsional rigidity of a length of a wire is obtained from the formula $N = \dfrac{8\pi I l}{t^2 r^4}$. If l is decreased by 2%, r is increased by 2%, t increased by 1.5% show that the value of N is diminished by 13% approximately.

(d) Find the nth derivative of $\dfrac{2x+3}{x^2+4x+3}$.

(e) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

(f) Find the eigen values and eigen vectors of

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

(g) Find half range sine series of

$f(x) = 3x - 2$ in $0 < x < \measuredangle$.

(h) Find the half range cosine series of $f(x) = e^x$ in $0 < x < \pi$.

(8 x 5 = 40 Marks)

II (a) (i) Find the radius of curvature of any point of the curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$. (8)

(ii) If $u = \log\left(\dfrac{x^5 + y^5 + z^5}{x^2 + y^2 + z^2}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 3u$. (7)

(Or)

(b) (i) Discuss the maxima and minima of $x^2 y^2 (1 - x - y)$. (8)

(ii) Find the evolute of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. (7)

III (a) (i) Test the convergence of the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\ldots$$ (8)

(ii) If $y=\left(\sin^{-1}x\right)^2$ Prove that $\left(1-x^2\right)y_{n+2}-\left(2n+1\right)xy_{n+1}-n^2y_n=0$. (7)

(Or)

(b) (i) For what value of x the series converges $x-\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}-\frac{x^4}{\sqrt{4}}+\ldots\ldots..$ (8)

(ii) Test the convergence of the series $\sum\frac{\left(1+nx\right)^n}{n^n}$. (7)

IV (a) (i) Test the consistency and solve $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$. (8)

(ii) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{pmatrix}$ (7)

(Or)

(b) (i) Verify Cayley – Hamilton theorem for $A=\begin{bmatrix}1 & 2\\ -1 & 3\end{bmatrix}$. Hence find A^{-1}. (8)

(ii) Solve by matrix method the equations $x+y+z=6$, $x-y+2z=5$, $3x+y+z=8$. (7)

V (a) (i) Expand $f\left(x\right)=x\sin x$ in $-\pi<x<\pi$ in Fourier series. (8)

(ii) Expand $f\left(x\right)=e^x$ as a cosine series in $0<x<l$. (7)

(Or)

(b) (i) Find the half range sine series for $f\left(x\right)=\begin{cases}\frac{1}{4}-x, & 0<x<\frac{1}{2}\\ x-\frac{3}{4}, & \frac{1}{2}<x<1\end{cases}$ (8)

(ii) Find the Fourier series expansion of $f\left(x\right)=4-x^2$, $-2\le x\le 2$. (7)

(4 x 15 = 60 Marks)

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