D 32976 (Pages : 2) Name.............................

Reg. No..............................

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2012

CS /IT 04 303—DISCRETE COMPUTATIONAL STRUCTURES

(2004 Admissions)

Time : Three Hours Maximum : 100 Marks

Part B

1. 1 Prove or disprove each of the following, where p, q and r are any statements.

(a) $\left[(p \wedge q) \wedge r\right] \Leftrightarrow \left[p \vee (q \vee r)\right]$.

(b) $\left[p \vee (q \rightarrow r)\right] \Leftrightarrow \left[(p \vee q) \rightarrow (p \vee r)\right]$.

2 Express the negation of the statement $p \Leftrightarrow q$ in terms of the connectives $\wedge$ and $\vee$.

3 Let $|A| = 5$. (a) How many directed graphs can one construct on A ? (b) How many of the graphs in part (c) Are actually undirected ?

4 If the complete graph Kn has 703 edges, how many vertices does it have ?

5 Draw all non-isomorphic, cycle-free, connected graphs having six vertices.

6 Determine $|V|$ for the following graphs or multigraphs G. (a) G has nine edges and all vertices have degree 3 ; (b) G is regular with 15 edges ; (c) G has 10 edges with two vertices of degree 4 and all others of degree 3.

7 Prove that a unit in a ring R cannot be a proper divisor of zero.

8 If R is a field, how many ideals does R have ?

($8 \times 5 = 40$ marks)

Part B

II. (a) (i) What is wrong with the following argument ?

Let A be a set with R a relation on A. If R is symmetric and transitive, then R is reflexive.

Proof : Let $(x, y) \in R$. By the symmetric property, $(y, x) \in R$. Then with (x, y), $(y, x) \in R$, it follows by the transitive property that $(x, x) \in R$. Consequently, R is reflexive.

(7 marks)

(ii) Let A = {1, 2, 3, 4, 5, 6, 7}. How many symmetric relations on A contain exactly (a) four ordered pairs ? ; (b) Five ordered pairs ? ; (c) Seven ordered pairs ? ; (d) Eight ordered pairs ?

(8 marks)

Or

Turn over

(b) (i) Show that $s(x) = x_2 + 1$ is reducible in $Z_2[x]$. (7 ½ marks)

(ii) Is $Z_2[x]/(s(x))$ an integral domain ? (7 ½ marks)

III. (a) Let R and S be relations from set A to B. Prove the following :

1. If $R \subseteq S$, then $S-1 \subseteq R-1$

2. If $R \subseteq S$, then $\overline{S} \subseteq \overline{R}$ ($\overline{S}$ is complements of S).

(7½ + 7½ = 15 marks)

Or

(b) If R is a relation on set A = { 1, 2, 3, 4} and R = { (1, 2) (2, 2) (2, 3) (3, 1) (3, 3) (4, 1)} Find out the following :

1 Reflexive closure of R.

2 Symmetric closure of R.

(7½ + 7½ = 15 marks)

IV. (a) (i) If f is a homomorphism from a commutative semi group (S, *) onto a semi group (T, *′), then show that (T, *′) is also commutative.

(ii) Define an ordered rooted tree. Cite any two applications of the tree structure, also illustrate using an example each the purpose of the usage.

(7½ + 7½ = 15 marks)

Or

(b) (i) Define a Hamilton path. Determine if the following graph has a Hamilton circuit.

(ii) Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 20 computers can directly access 20 different printers. Justify your answer.

(7½ + 7½ = 15 marks)

V. (a) (i) Let A = {1, 2, 4, 8, 16} and relation R1 be partial order of divisibility on A. Let A′ = {0, 1, 2, 3, 4} and R2 be the relation "less than or equal to" on integers. Show that ( ) ( ) A, R1 and A′, R2 are isomorphic posets.

(ii) Find a generating function to count the numbers of integral solutions to $e_1\ e_2\ e_3 = 10$, if for each, $l\ 0\ i \leq e$.

Or

(b) (i) Determine the number of integral solutions of the equation

$X_1 + X_2 + X_3 + X_4 = 20$

where

$2 \leq X_1 \leq 6,\ 3 \leq X_2\ 7,\ 5 \leq X_3\ 8,\ 2 \leq X_4 \leq 9$

(9 marks)

(ii) Find the generating function to represent the number of ways the sum 9 can be obtained when 2 distinguishable fair dice are tossed and the first shows an even number and the second shows an odd number.

(6 marks)

[4 × 15 = 60 marks]