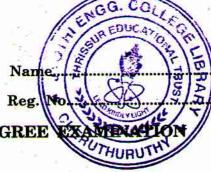
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THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE OCTOBER 2012

EN 09 301—ENGINEERING MATHEMATICS—III

(2009 Admissions)

[Common to all Branches]

Time: Three Hours

Maximum: 70 Marks

Part A

Answer all questions.

- 1. Determine constant 'a' such that $u = e^{ax} \cos 3y$ is harmonic.
- 2. Discuss the nature and location of singularities of the function $f(z) = \frac{\tan z}{z}$.
- 3. Find the critical points if any of the mapping $W = \sin z$.
- 4. Show that the set of all 2×2 non-singular matrices is not a vector space.
- 5. Find the Fourier transform of the function $f(t) = \begin{cases} 5, & -2 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$

 $(5 \times 2 = 10 \text{ marks})$

Part B

Answer any four questions.

- 6. Show that $|z|^2$ is not analytic at any point.
- 7. Evaluate $\int_{0}^{2+i} (\overline{z})^2 dz$ along the line $y = \frac{x}{2}$.
- 8. Find the Fourier sine transform of $3e^{-2x} + 2e^{-3x}$.
- 9. If F(s) is the Fourier transform of f(x), then show that $F\{f(ax)\} = \frac{1}{a}F(\frac{S}{a})$.

10. Is
$$U_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
, $U_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $U_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ a basis of \mathbb{R}^3 .

11. If W be a proper sub-space of a finite dimensional vector space V, then show that W is finite dimensional and dim W ≤ dim V.

 $(4 \times 5 = 20 \text{ marks})$

Part C

Answer all questions as per choice given.

- 12. (a) Determine the analytic function f(z) = u + iV where $u + V = (x y)(x^2 + 4xy + y^2)$.

 Or
 - (b) Find the image of the first quadrant x > 0, y > 0 under $W = \frac{z i}{z + i}$.
- 13. (a) (i) Find the Laurent's series expansion of $\frac{e^{2z}}{(z-1)^3}$ about the singularity z=1.
 - (ii) Evaluate $\int_{C} \frac{dz}{\left(z^2+4\right)^2}$; C is |z-i|=2.

Or

- (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)(x^2 + 4)}$
- 14. (a) Find a basis and the dimension of the subspace W of R4 generated by

$$(1, -4, 1, 3), (2, -1, 3, -1)$$
 and $(0, 2, 1, -5)$.

Or

- (b) Apply Gram-Schmidth process to the vectors $\beta_1 = (1, 0, 1)$, $\beta_2 = (1, 0, -1)$, $\beta_3 = (0, 3, 4)$ to obtain an orthonormal basis for \mathbb{R}^3 with the standard inner product.
- 15. (a) Find the Fourier Cosine transform of $f(x) = \frac{1}{1+x^2}$.

Or

(b) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate

$$\int_{0}^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda.$$

 $(4 \times 10 = 40 \text{ marks})$