D 30886 (Pages : 2) Name.....

Reg. No.....

THIRD SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION OCTOBER 2012

EN 09 301—ENGINEERING MATHEMATICS—III

(2009 Admissions)

[Common to all Branches]

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Determine constant 'a' such that $u = e^{ax}\cos 3y$ is harmonic.
2. Discuss the nature and location of singularities of the function $f(z)=\dfrac{\tan z}{z}$.
3. Find the critical points if any of the mapping $W = \sin z$.
4. Show that the set of all 2×2 non-singular matrices is not a vector space.
5. Find the Fourier transform of the function $f(t)=\begin{cases}5, & -2\le t\le 2\\ 0, & \text{otherwise}\end{cases}$

($5 \times 2 = 10$ marks)

Part B

Answer any **four** *questions.*

6. Show that $|z|^2$ is not analytic at any point.
7. Evaluate $\int_0^{2+i}(\bar{z})^2\,dz$ along the line $y=\dfrac{x}{2}$.
8. Find the Fourier sine transform of $3e^{-2x}+2e^{-3x}$.
9. If $F(s)$ is the Fourier transform of $f(x)$, then show that $F\{f(ax)\}=\dfrac{1}{a}F\left(\dfrac{S}{a}\right)$.
10. Is $U_1=\begin{bmatrix}1\\1\\1\end{bmatrix}$, $U_2=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $U_3=\begin{bmatrix}2\\-1\\1\end{bmatrix}$ a basis of $\mathbb{R}^3$.

Turn over

11. If W be a proper sub-space of a finite dimensional vector space V, then show that W is finite dimensional and dim W $\leq$ dim V.

(4 × 5 = 20 marks)

Part C

Answer **all** *questions as per choice given.*

12. (a) Determine the analytic function $f(z)=u+iV$ where $u+V=(x-y)\left(x^2+4xy+y^2\right)$.

Or

(b) Find the image of the first quadrant $x>0, y>0$ under $W=\dfrac{z-i}{z+i}$.

13. (a) (i) Find the Laurent's series expansion of $\dfrac{e^{2z}}{(z-1)^3}$ about the singularity $z=1$.

(ii) Evaluate $\displaystyle\int_C \frac{dz}{\left(z^2+4\right)^2}$; C is $|z-i|=2$.

Or

(b) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x^2 dx}{\left(x^2+1\right)\left(x^2+4\right)}$.

14. (a) Find a basis and the dimension of the subspace W of $\mathbb{R}^4$ generated by

$(1,-4,1,3),(2,-1,3,-1)$ and $(0,2,1,-5)$.

Or

(b) Apply Gram-Schmidth process to the vectors $\beta_1=(1,0,1),\beta_2=(1,0,-1),\beta_3=(0,3,4)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product.

15. (a) Find the Fourier Cosine transform of $f(x)=\dfrac{1}{1+x^2}$.

Or

(b) Express the function $f(x)=\begin{cases}1 & \text{for } |x|\leq 1\\ 0 & \text{for } |x|>1\end{cases}$ as a Fourier integral. Hence evaluate

$\displaystyle\int_0^{\infty} \frac{\sin\lambda\cos\lambda x}{\lambda} d\lambda$.

(4 × 10 = 40 marks)