C 28763 (Pages : 2) Name.........................

Reg. No.........................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2012

CS 04 604—GRAPH THEORY AND COMBINATORICS

Time : Three Hours Maximum : 100 Marks

Part A

I. (a) Explain the Chinese postman problem with an example.

(b) Can a bipartite graph contain a cycle of odd length ? Explain.

(c) How many leaves does a full binary tree have if its height is :

(i) 3 ? (ii) 12 ?

(d) Write the pseudocode for the Prim's Algorithm and explain.

(e) How many permutations of size 3 can one produce with the letters m, r, a, f and t ?

(f) If eight distinct dice are rolled, what is the probability that all *six* numbers appear ?

(g) Solve the following recurrence relations.

(i) $a_{n+2} - a_n = \sin(n\pi / 2), n \geq 0\ a_0 = 1, a_1 = 1.$

(h) Write a note on the usage of the summation operator.

(8 × 5 = 40 marks)

Part B

II. (a) Let G = (V, E) be a loop-free undirected graph that is 6-regular. Prove that if $|V| = 1,$ then G contains a Hamilton cycle.

Or

(b) Explain in detail about the graph colouring and chromatic polynomials with examples.

III. (a) (i) A complete ternary (or 3-ary) tree T = (V, E) has 34 internal vertices. How many edges does T have ? How many leaves ?

(7 marks)

(ii) How many internal vertices does a complete 5-ary tree with 817 leaves have ?

(8 marks)

Or

(b) Explain the max-flow min-cut theorem with an example.

(15 marks)

Turn over

IV. (a) In how many ways can we distribute eight identical white balls into four distinct-containers so that (i) no container is left empty ? (ii) the fourth container has an odd number of balls in it ?

Or

(b) If $n \in Z^+$, prove that :

(i) $\phi(2n) = 2\phi(n)$ when n is even and

(ii) $\phi(2n) = \phi(n)$ when n is odd.

V. (a) (i) Find the generating function for the number of ways to select 10 candy bars from large supplies of six different kinds.

(7 marks)

(ii) Find the generating function for the number of ways to select, with repetitions allowed, r objects from a collection of Ω distinct objects.

(8 marks)

Or

(b) (i) If $a_0 = 0$, $a_1 = 1$, $a_2 = 4$ and $a_3 = 37$ satisfy the recurrence relation, $a_{n+2} + ba_{n+1} + ca_n = 0$, where $n \geq 0$ and b, c are constants, solve for a_n.

(8 marks)

(ii) Write a note on the exponential generating function.

(7 marks)

[4 × 15 = 60 marks]