

C 28763

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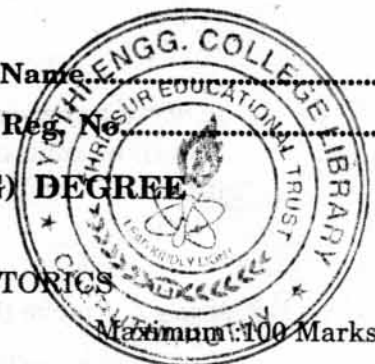
Name.....

Reg. No.....

**SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE
EXAMINATION, JUNE 2012**

CS 04 604—GRAPH THEORY AND COMBINATORICS

Time : Three Hours



Part A

- I. (a) Explain the Chinese postman problem with an example.
(b) Can a bipartite graph contain a cycle of odd length? Explain.
(c) How many leaves does a full binary tree have if its height is :
(i) 3? (ii) 12?
(d) Write the pseudocode for the Prim's Algorithm and explain.
(e) How many permutations of size 3 can one produce with the letters m, r, a, f and t ?
(f) If eight distinct dice are rolled, what is the probability that all six numbers appear?
(g) Solve the following recurrence relations.
(i) $a_{n+2} - a_n = \sin(n\pi/2), n \geq 0, a_0 = 1, a_1 = 1.$
(h) Write a note on the usage of the summation operator.

(8 × 5 = 40 marks)

Part B

- II. (a) Let $G = (V, E)$ be a loop-free undirected graph that is 6-regular. Prove that if $|V| = 1$, then G contains a Hamilton cycle.

Or

- (b) Explain in detail about the graph colouring and chromatic polynomials with examples.

- III. (a) (i) A complete ternary (or 3-ary) tree $T = (V, E)$ has 34 internal vertices. How many edges does T have? How many leaves?

(7 marks)

- (ii) How many internal vertices does a complete 5-ary tree with 817 leaves have?

(8 marks)

Or

- (b) Explain the max-flow min-cut theorem with an example.

(15 marks)

Turn over

- IV. (a) In how many ways can we distribute eight identical white balls into four distinct containers so that (i) no container is left empty ? (ii) the fourth container has an odd number of balls in it?

Or

- (b) If $n \in \mathbb{Z}^+$, prove that :

(i) $\phi(2n) = 2\phi(n)$ when n is even and

(ii) $\phi(2n) = \phi(n)$ when n is odd.

- V. (a) (i) Find the generating function for the number of ways to select 10 candy bars from large supplies of six different kinds.

(7 marks)

- (ii) Find the generating function for the number of ways to select, with repetitions allowed, r objects from a collection of Ω distinct objects.

(8 marks)

Or

- (b) (i) If $a_0 = 0, a_1 = 1, a_2 = 4$ and $a_3 = 37$ satisfy the recurrence relation, $a_{n+2} + ba_{n+1} + ca_n = 0$, where $n \geq 0$ and b, c are constants, solve for a_n .

(8 marks)

- (ii) Write a note on the exponential generating function.

(7 marks)

[4 × 15 = 60 marks]