C 26470 (Pages : 3) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, APRIL 2012

PTEN/EN09 101—ENGINEERING MATHEMATICS—I

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Give the formula for circle of curvature in Cartesian form.
2. What is Raabe's test ? Define.
3. Find the eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.
4. Test whether $f(x) = |\cos x|$ is odd function or even function in the interval $(-\pi, \pi)$.
5. By using the sine series for $f(x) = 1$ in $0 < x < \pi$, show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots$

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^{2n}$.
7. If "ρ" is the radius of curvature at any point (x, y) on the curve $y = \frac{ax}{a+x}$, show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$.
8. Give the extreme values of the function $f(x, y) = x^3 y^2 (12 - x - y)$.
9. Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Turn over

10. Verify Cayley–Hamilton theorem for matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$.

11. Expand $f(x) = \frac{1}{4} - x$, if $0 < x < \frac{1}{2}$

$= x - \frac{3}{4}$, if $\frac{1}{2} < x < 1$ as the Fourier series of sine terms.

(4 × 5 = 20 marks)

Part C

Answer section (a) **or** *section (b) of each question.*

12. (a) Find $\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}}$ if $x^2 + y^2 + z^2 - 2xyz = 1$.

Or

(b) Express $\iiint \sqrt{xyz(1-x-y-z)}\,dx\,dy\,dz$ in terms of u, v, w given that $x + y + z = u$, $y + z = uv$, $z = uvw$.

13. (a) Discuss the convergence of the series $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \ldots\ldots\infty$

Or

(b) Test the series for conditional convergence

$$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3)$$
$$-\frac{1}{5^3}(1+2+3+4) + \ldots\ldots. + \infty.$$

14. (a) Reduce the quadratic form $2x_1^2 + 5x_2^2 + 3x_3^2 + 4x_1 x_2$ to canonical form by an orthogonal Transformation.

Or

(b) Find the eigenvalues of A and hence find A^n ("n" is a positive integer), given that $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

15. (a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$.

Or

(b) Show that for $(-\pi < x < \pi)$, $\sin ax = \frac{2\sin(a\pi)}{\pi}$

$$\left(\frac{\sin x}{1^2 - a^2} - \frac{2\sin 2x}{2^2 - a^2} + \frac{3\sin 3x}{3^2 - a^2} - \ldots\ldots\right).$$

(4 × 10 = 40 marks)