D 23473 (Pages : 2) Name.............................

Reg. No.............................

FIFTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION DECEMBER 2011

IT 04 505—GRAPH THEORY AND COMBINATORIES

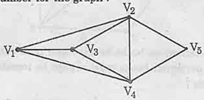
(2004 Admissions)

Time : Three Hours Maximum : 100 Marks

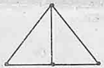
Part A

I. (a) Define (i) Path ; (ii) Cycle.

(b) Find the chromatic number for the graph :



(c) Find all the spanning trees of the graph :



(d) Explain Bellman-Ford algorithm.

(e) Find the number of permutations of the letters of the word SUCCESS.

(f) Discuss principle of inclusion-exclusion for two sets, with an example.

(g) Find the generating function for the sequence :

$a_r = 1$ for $0 \le r \le n$ and $a_r = 0$ for $r \ge n+1$.

(h) Solve the recurrence relation $a_n = 7\, a_{n-1}$, where $n \ge 1$, given that $a_2 = 98$.

(8 × 5 = 40 marks)

Part B

II. (a) A simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges.

(15 marks)

Or

(b) (i) Prove that the complete bipartite graph $K_{2,3}$ contains an Euler trail. (7 marks)

(ii) Prove that the Petersen graph contains neither an Euler circuit nor an Euler trail.

(8 marks)

Turn over

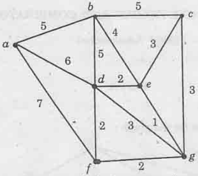
IT

III. (a) (i) Prove that a graph is connected iff it has a spanning tree. (7 marks)

(ii) Discuss about Prim's algorithm. (8 marks)

Or

(b) Explain about Kruskal's algorithm and apply Kruskal's algorithm to the weighted graph.



(15 marks)

IV. (a) (i) In how many ways can seven books be arranged on a shelf is (1) any arrangement is allowed ; (2) Three particular books must always be together ; (3) Two particular books must occupy the ends ?

(9 marks)

(ii) Find the number of distinguishable permutations of the letters in the following words : (a) BASIC ; (b) PEPPER.

(6 marks)

Or

(b) (i) Evaluate d_5, d_6, d_7, d_8 where d_n is the number of derangements of n objects. (12 marks)

(ii) State Pigeon-hole principle andits generalization. (3 marks)

V. (a) Find the sequences generated by the following functions :—

(i) $(3+x)^3$.

(ii) $2x^2(1-x)^{-1}$.

(iii) $\frac{1}{1-x}+2x^3$.

(iv) $(1+3x)^{-1/3}$.

(v) $3x^3+e^{2x}$.

(15 marks)

Or

(b) Solve the recurrence relations :

(i) $a_n = 3a_{n-1} - 2a_{n-2}, n \geq 2, a_1 = 5$ and $a_2 = 3$. (8marks)

(ii) $a_n - 6a_{n-1} + 9a_{n-2} = 0, n \geq 2, a_0 = 5, a_1 = 12$. (7 marks)

[4 × 15 = 60 marks]