C 18255 (Pages : 2) Name.................................

Reg. No...............................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2011

ME 04 605—OPERATIONS RESEARCH

Time : Three Hours Maximum : 100 Marks

Answer all *questions.*

I. (a) Find the rank of the matrix $\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$.

(b) Show that the vectors (3, 1, – 4), (2, 2, – 3), (0, – 4, 1) and (– 4, – 4, 6) are linearly dependent. Find the relationship between them.

(c) Write down the dual LP problem for the following LP problem :—

$$\text{Max. } Z = 8x_1 + 2x_2 + 20x_3$$

$$\begin{aligned} \text{subject to} \quad x_1 + x_2 - x_3 &\le 10 \\ 2x_1 + x_2 + 10x_3 &\le 90 \\ 3x_1 + x_3 &\le 10 \\ 4x_1 + x_2 + 4x_3 &\ge 25 \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

(d) Briefly explain two-phase method.

(e) Compare stepping stone method with UV-method.

(f) Explain the LP representation of a game theory problem using a simple example.

(g) Describe briefly the queueing model with Poisson arrival and constant service time.

(h) What is meant by priority queues ? Explain with an example.

(8 × 5 = 40 marks)

II. (a) Find for what value of k, the set of equations $2x - 3y + 6z - 5w = 3$, $y - 4z + w = 1$, $4x - 5y + 8z - 9w = k$ has (i) no solution ; (ii) infinitely many solution.

Or

(b) Show that the vectors (1, – 1, – 1, 3), (2, 1, – 2, – 1), (1, 0, 2, 1) and (0, 1, 1, 0) are linearly independent. Express the vector (1, 1, 1, 1) as a linear combination of the above vectors.

Turn over

III. (a) Solve the following LP problem by simplex method :—

$$\text{Max. } Z = 10x_1 + 16x_2 + 8x_3$$

subject to

$$x_1 + 2x_2 + x_3 \leq 20$$
$$2x_1 + x_2 + 3x_3 \leq 30$$
$$x_1, x_2, x_3 \geq 0.$$

Or

(b) Solve the following LP problem by Charne's M-method :—

$$\text{Min. } Z = x_1 + 3x_2 + 2x_3$$

subject to

$$2x_1 + x_2 - x_3 \geq 5$$
$$x_1 + 2x_2 + x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0.$$

IV. (a) Solve the following transportation problem by stepping stone method :

Mine \ Industry	X	Y	Z	Availability/month
A	5	10	15	50
B	20	25	15	100
C	10	8	15	50
Requirement/month	50	70	80	

Or

(b) Solve the following two person zerosum game graphically :

Player B

$$\text{Player A} \begin{pmatrix} 10 & 5 & -1 & 2 & -5 & 7 \\ 2 & 10 & 5 & 8 & 2 & -3 \end{pmatrix}$$

V. (a) Customers arrive at a shop in a Poisson process with mean interarrival time of 3 minutes. There is only one server in the shop and the service times are exponentially distributed with mean value of 150 seconds. Find (i) the average number of customers in the shop ; (ii) the mean number of customers in the waiting time ; (iii) the average waiting time of a customer ; (iv) percentage of time the queue is empty ; (v) probability that an arriving customer finds at least 3 persons in the shop.

Or

(b) Find the optimum solution of the following problem using dynamic programming technique :—

$$\text{Maximize } Z = x_1 x_2 x_3$$

subject to

$$2x_1 + x_2 + 3x_3 = 50$$
$$x_1, x_2, x_3 \geq 0.$$

(4 × 15 = 60 marks)