

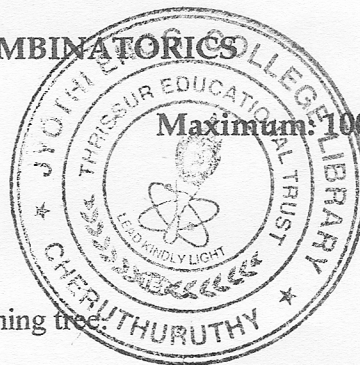
FIFTH SEMESTER B.TECH (ENGINEERING) DEGREE EXAMINATION,
DECEMBER 2010

IT 04 505 - GRAPH THEORY AND COMBINATORICS

Time: 3 Hours

Maximum: 100 Marks

Answer all questions



1. (a) State and prove Euler's formula for planar graphs.
 (b) Compare Kruskal's and Prim's algorithms for minimal spanning tree.
 (c) State the max-flow min-cut theorem.
 (d) Using generating functions, compute
 $1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$
 (e) Define a path, walk and cycles in graph.
 (f) Write the Bellman Ford algorithm.
 (g) How many 5 letter words can be formed from the word PARIPASSU?
 (h) Find the generating function of $1!, 2!, 3!, \dots$

(8 × 5 = 40)

2. (a) Show that the maximum flow between any two vertices is equal to minimum of the capacities of all cut-sets with these two vertices.

OR

- (b) State and prove max-flow-min-cut theorem
3. (a) Solve the recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 2, a_0 = 2, a_1 = -8$.

OR

- (b) Find the generating function of recurrence relation $a_n = 4a_{n-1}; a_0 = 1$.
4. (a) State and prove 5-colour theorem for planar graphs.

OR

- (b) Prove that a graph G with n vertices has a Hamiltonian path if the sum of the degrees of every pair of vertices v_i, v_j in G satisfies the condition.

$$d(v_i) + d(v_j) \geq n - 1$$

5. (a) Let a_n denote the number of ways of computing the product of n matrices. For example, the product of 3 matrices ABC could be computed in 2 ways : (AB)C or A (B C).

Prove that $a_n = \frac{(2n-2)!}{n!(n-1)!}$

OR

- (b) Determine the number of ways of placing $2t + 1$ distinguishable balls in three distinct boxes so that any two boxes together will contain more balls than the other one.

(15 × 4 = 60)
