C 14941 (Pages : 2) Name.............................

Reg. No.............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, DECEMBER 2010

EN 2K 101—MATHEMATICS—I

(Common to all Branches)

Time : Three Hours Maximum : 100 Marks

Answer all *questions.*

Part A

I. 1 Evaluate $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$.

2 If u is a homogeneous function of degree n in x and y show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$$

3 Discuss the convergence of $\sum \frac{(1+nx)^n}{n^n}$.

4 Show that $e^{\sin x} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} + \ldots$

5 Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$.

6 Determine the values of λ for which the equations $3x_1 + x_2 + \lambda x_3 = 0$; $4x_1 + 2x_2 + 3x_3 = 0$; $2\lambda x_1 + 4x_2 + \lambda x_3 = 0$ have a non-trivial solution. Find the solution.

7 Express $f(x) = |x|$, $-\pi < x < \pi$ as Fourier series.

8 Obtain a half range sine series for :

$$f(x) = kx \quad \text{for } 0 \le x \le l/2$$
$$= k(l-x) \quad \text{for } l/2 \le x \le l$$

(8 × 5 = 40 marks)

Part B

Module I

II. A (a) Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(b) If $v = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.

Or

Turn over

B (a) Using Lagranges method of multipliers determine the dimensions of the rectangular parellelopiped of largest possible volume with faces parallel to co-ordinate planes and which can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(b) If $r = {yz}/{x}$; $s = {zx}/{y}$; $t = {xy}/{z}$ show that $J\left(\frac{r, s, t}{x, y, z}\right) = 4$.

(1 × 15 = 15 marks)

MODULE II

III. A Using Leibnitz's theorem. Obtain the Maclaurin's series expansion of $\tan^{-1} x$.

Or

B Discuss the convergence of the following series :—

(i) $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \ldots\ldots$

(ii) $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \ldots\ldots (0 < x < 1)$.

(1 × 15 = 15 marks)

MODULE III

IV. A Reduce $3x^2 + 5y^2 + 3z^2 - 2yz - 12zx - 2xy$ to canonical form. Specify the nature.

Or

B (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

(b) Find the values of λ and μ such that $x + 2y + \lambda z = 1$; $x + 2\lambda y + z = \mu$; $\lambda x + 2y + z = 1$ have (i) no solution ; (ii) unique solution ; and (iii) many solutions.

(1 × 15 = 15 marks)

MODULE IV

V. A (a) Find the Fourier series of period $2l$ for the function $f(x) = \begin{cases} l - x, & 0 \le x \le l \\ 0, & l \le x \le 2l \end{cases}$.

Hence find the sum of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$

(b) Show that in the range, $0 \le x \le \pi$,

$$x(\pi - x) = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \ldots\ldots\right).$$

Or

B The following values of x and y are given. Expand y in the form of a Fourier series upto second harmonic :

x	:	0	1	2	3	4	5	6
y	:	90	182	444	278	275	220	90

(1 × 15 = 15 marks)