C 15004 (Pages : 2) Name.................................

Reg. No.................................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2011

EN 09 102/PTEN 09—ENGINEERING MATHEMATICS—II

(2009 admissions)

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*
Each question carries 2 *marks.*

1. Solve $\left(D^4 - 2D^3 + D^2\right)_y = 0$.

2. If $L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\} = \frac{t}{2}\sin t$, find $L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$.

3. Find ϕ, if grad $\phi = \left(y^2 - 2xyz^3\right)\hat{i} + \left(3 + 2xy - x^2z^3\right)\hat{j} + \left(6z^3 - 3x^2yz^2\right)\hat{k}$.

4. Find the equation of the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point (1, – 1, 2).

5. State Green's theorem.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*
Each question carries 5 *marks.*

6. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is the parameter.

7. Solve $\left(x^2 + y^2 - a^2\right)x\,dx + \left(x^2 - y^2 - b^2\right)y\,dy = 0$.

8. Find using convolution theorem to the inverse Laplace transforms of $\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$.

9. Find div $\vec{F}$ and curl $\vec{F}$, where $\vec{F} = \text{grad}\left(x^3 + y^3 + z^3 - 3xyz\right)$.

10. Find the area of a circle of a radius a using Green's theorem.

Turn over

11. Evaluate $\iint_S \vec{F} \cdot \overrightarrow{dS}$, where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 1$ that lies in the first Octant.

(4 × 5 = 20 marks)

Part C

Answer Section (a) or (b) of each question.
Each question carries 10 marks.

12. (a) Solve $(D^2 + a^2)_y = \sec ax$.

Or

(b) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + a^2 y = \sec ax$.

13. (a) Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{s^2}\right)$.

Or

(b) Find the Laplace transform of $L\left\{\frac{e^{at} - \cos bt}{t}\right\}$.

14. (a) If $r = |\vec{r}|$, where $\vec{r}$ is the position vector of the point (x, y, z) with respect of the origin, prove that $\nabla f(r) = \frac{f'(r)}{r}\bar{r}$.

Or

(b) Find the constants a, b, c, so that $\bar{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ may be irrotational.

15. (a) Use divergence theorem to evaluate $\iint_S \left(yz^2\hat{i} + 2x^2\hat{j} + 2z^2\hat{k}\right) \cdot dS$, where $\bar{S}$ is the closed surface bounded by the xoy-plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ above this plane.

Or

(b) If S is any closed surface enclosing a volume V and if $\bar{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$ prove that $\iint_S \bar{A} \cdot d\bar{S} = (a + b + c)\,V$.

(4 × 10 = 40 marks)