(AC1)

C 15003 (Pages : 3) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2011

PTEN/EN 09 101—ENGINEERING MATHEMATICS—I

(2009 admissions)

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*
Each question carries 2 marks.

1. Define Convergent and Divergent sequence.
2. Define radius of curvature in Cartesian co-ordinates.
3. The sum of the eigenvalues of a matrix A is equal to the sum ——— elements of a given square matrix A.
4. Give the Fourier series for the function $f(x)$ in the interval $\alpha < x < \alpha + 2\pi$, giving the definition of Euler's formulae.
5. Given that $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, find its eigenvalue.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*
Each question carries 5 marks.

6. Test for convergence the series :

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots \infty.$$

7. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$.

8. Discuss the convergence of the following series :—

$$1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots + \infty.$$

9. Find the Taylor's series expansion of $e^x \sin y$ near the point $\left(-1, \frac{\pi}{4}\right)$ upto the third degree terms.

Turn over

10. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

11. Find a Fourier series to represent x^2 in the interval $(-l, l)$.

(4 × 5 = 20 marks)

Part C

Answer Section (a) or Section (b) of each question.
Each question carries 10 marks.

12. (a) Verify that the eigenvectors of the real symmetric matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are orthogonal in pairs.

Or

(b) Verify that the eigenvalues of A^2 and A^{-1} are respectively the squares and reciprocals of the eigenvalues of A, given that $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$.

13. (a) Test for convergence the series using Raabe's test :

$$\sum \frac{4 \cdot 7 \cdots (3n+1)}{1 \cdot 2 \cdots n} x^n.$$

Or

(b) Examine the character of the series :

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{2n-1}$ and

(ii) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1} x^n}{n(n-1)}, 0 < x < 1.$

14. (a) Obtain the Fourier series for

$$f(x) = \frac{1}{4} - x, \quad \text{if } 0 < x < \frac{1}{2}$$

$$= x - \frac{3}{4}, \quad \text{if } \frac{1}{2} < x < 1.$$

Or

(b) Obtain the first 3 coefficients in the Fourier cosine series for y, where y is given in the following table :—

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

15. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and also find A^{-1}.

Or

(b) Diagonalise the matrix $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ by means of an orthogonal transformation.