C 14974 (Pages 2) Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2011

EN 04 102—MATHEMATICS—II

Time : Three Hours Maximum : 100 Marks

Part A

1. (a) Find the particular integral of $(D^3 + 1)\, y = \cos(2x - 1)$.

(b) Solve $p^2 + 2py \cot x = y^2$.

(c) Find the inverse Laplace transforms of $\dfrac{s}{(s^2 + a^2)^2}$.

(d) Find the Laplace transforms of $\cos^2(2t)$.

(e) Evaluate div $(3x^2 i + 5xy^2 j + xyz^3 k)$ at the point (1, 2, 3).

(f) If $F = (x + y + 1)\, i + j - (x + y)\, k$, show that F.curl F = 0.

(g) Evaluate the line integral $\int_C \left[(x^2 + xy)dx + (x^2 + y^2)\, dy \right]$ where C is the square formed by the lines $y = \pm 1$ and $x = \pm 1$.

(h) Evaluate $\int_S (yzi + zxj + xyk) \cdot ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant.

(8 × 5 = 40 marks)

Part B

2. (a) (i) Solve $(D^2 - 1)\, y = x \sin 3x + \cos x$. (8 marks)

(ii) Solve $xy\left(1 + xy^2\right)\dfrac{dy}{dx} = 1$. (7 marks)

Or

(b) (i) Solve $\dfrac{d^2y}{dx^2} + a^2 y = \sec ax$. (8 marks)

Turn over

(ii) Solve $\frac{y}{x}\frac{dy}{dx} + \frac{x^2 + y^2 - 1}{2(x^2 + y^2)^{+1}} = 0.$ (7 marks)

3. (a) (i) Find the inverse Laplace transforms of $\frac{s+2}{s^2(s+1)(s-2)}$. (8 marks)

(ii) Find the Laplace transform of $\left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^3$. (7 marks)

Or

(b) (i) Find the Laplace transform of $\frac{1}{2}\log\left(\frac{s^2 + b^2}{s^2 + a^2}\right)$. (8 marks)

(ii) Find the inverse Laplace transform of $\log\frac{(s+1)}{(s+2)(s+3)}$. (7 marks)

4. (a) (i) Show that div (grad r^n) = $n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. (8 marks)

(ii) Find curl (curl A) given $A = x^2y\,I + y^2z\,J + z^2y\,k$. (7 marks)

Or

(b) (i) If $F = (x + y + 1)\,i + j - (x + y)\,k$ show that F.curl F = 0. (8 marks)

(ii) If $u = x^2yz, v = xy - 3z^2$, find $\nabla\cdot(\nabla u \times \nabla v)$. (7 marks)

5. (a) (i) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$. (8 marks)

(ii) If $uF = \nabla v$, where u, v are scalar fields and F is a vector, show that $\vec{F}\cdot\text{curl}\,\vec{F} = 0$. (7 marks)

Or

(b) Evaluate $\int_C \left[(x^2 + xy)\,dx + (x^2 + y^2)\,dy\right]$, where C is the square formed by the lines $x = \pm 1, y = \pm 1$. (15 marks)

[4 × 15 = 60 marks]