

C 14973

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Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)
DEGREE EXAMINATION, MAY 2011

EN 04 101—ENGINEERING MATHEMATICS—I

(2004 admissions)

Time : Three Hours

Maximum : 100 Marks

Answer all questions.

Part A

I. (a) Evaluate $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{1/x}$

(b) If $u = \log(x^2 + y^3 + z^3 - 3xyz)$ prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = \frac{-9}{(x+y+z)^2}$.

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n^3 + 2}{2^n + 2} \right)$.

(d) Expand $\cos x$ in powers of $x - \frac{\pi}{4}$ upto 4 terms.

(e) Prove that the eigen values of a unitary matrix are of magnitude unity.

(f) Show that the equations $x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2, x - y + z = -1$ are consistent.

(g) Express x in Fourier series of period 2π , with $-\pi < x < \pi$.

(h) Obtain the Fourier series expansion of $f(x) = x^2 - 2$ in $-2 < x < 2$.

(8 × 5 = 40 marks)

Part B

II. (a) (i) Prove that the evolute of the tractrix $x = a \left(\cos t + \log \tan \frac{t}{2} \right), y = a \sin t$ is the catenary $y = a \cosh(x/a)$.

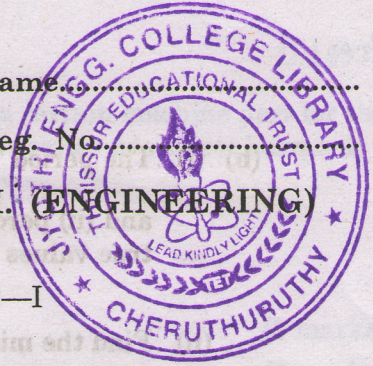
(8 marks)

(ii) In the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ show that the radius of curvature at the end of the major axis is equal to the semi latus rectum.

(7 marks)

Or

Turn over



(b) (i) The period T of a simple pendulum of length l is given by $T = 2\pi \sqrt{\frac{l}{g}}$. Find (i) the error and (ii) percentage error made in computing T by using $l = 2\text{ft}$ and $g = 32\text{ ft/sec.}^2$, if the true values are $l = 1.95\text{ ft.}$ and $g = 32.2\text{ ft/sec.}^2$

(8 marks)

(ii) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

(7 marks)

III. (a) (i) Discuss the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots$

(8 marks)

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{4.7 \dots (3n+1)}{n!} x^n \right)$.

(7 marks)

Or

(b) (i) If $y = (\sinh^{-1} x)^2$ show that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$.

(8 marks)

(ii) Test the convergence $\sum_{n=1}^{\infty} \left((-1)^n \frac{1+n^2}{1+n^3} \right)$.

(7 marks)

IV. (a) (i) Using Cayley-Hamilton theorem find A^{-1} for $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$.

(8 marks)

(ii) Find λ so that $\lambda(x^2 + y^2 + z^2) + 2xy - 2yz + 2zx$.

(7 marks)

Or

(b) Reduce the quadratic form $3x^2 - 2y^2 - z^2 + 12yz + 8zx - 4xy$ into canonical form by orthogonal reduction. Also identify the nature of the quadratic form.

(15 marks)

V. (a) (i) Obtain the Fourier series expansion for the function $f(x) = x^2, -\pi < x < \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

(8 marks)

(ii) Expand the function $f(x)$ defined by $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$ in Fourier series.

(7 marks)

Or

- (b) (i) The following values of y gives the displacement (in cm) of a certain machine part for the rotation x of the flywheel. Express y in the form of a Fourier series upon first harmonic :

$x :$	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$
$y :$	0	9.2	14.4	17.8	17.3	11.7

(8 marks)

- (ii) Find the half range sine series for $f(x) = \cos x$, $0 < x < \pi$.

(7 marks)

[4 × 15 = 60 marks]