C 14973 (Pages : 3) Name..............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2011

EN 04 101—ENGINEERING MATHEMATICS—I

(2004 admissions)

Time : Three Hours Maximum : 100 Marks

Answer all *questions.*

Part A

I. (a) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x}$

(b) If $u = \log\left(x^2+y^3+z^3-3xyz\right)$ prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n^3+2}{2^n+2}\right)$.

(d) Expand $\cos x$ in powers of $x-\frac{\pi}{4}$ upto 4 terms.

(e) Prove that the eigen values of a unitary matrix are of magnitude unity.

(f) Show that the equations $x+2y-z=3, 3x-y+2z=1, 2x-2y+3z=2, x-y+z=-1$ are consistent.

(g) Express x in Fourier series of period 2π, with $-\pi < x < \pi$.

(h) Obtain the Fourier series expansion of $f(x) = x^2 - 2$ in $-2 < x < 2$.

(8 × 5 = 40 marks)

Part B

II. (a) (i) Prove that the evolute of the tractrix $x = a\,(\cos t + \log\tan\frac{t}{2}), y = a\sin t$ is the catenary $y = a\cosh(x/a)$.

(8 marks)

(ii) In the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ show that the radius of curvature at the end of the major axis is equal to the semi latus rectum.

(7 marks)

Or

Turn over

(b) (i) The period T of a simple pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$. Find (i) the error and (ii) percentage error made in computing T by using $l = 2ft$ and $g = 32$ ft/sec.^{2}, if the true values are $l = 1.95$ ft. and $g = 32.2$ ft/sec.^{2}

(8 marks)

(ii) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$. (7 marks)

III. (a) (i) Discuss the convergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots$ (8 marks)

(ii) Test the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{4.7.\cdots(3n+1)}{n!}x^n\right)$. (7 marks)

Or

(b) (i) If $y = \left(\sinh^{-1} x\right)^2$ show that $\left(1+x^2\right)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$. (8 marks)

(ii) Test the convergence $\sum_{n=1}^{\infty}\left((-1)^2 \frac{1+n^2}{1+n^3}\right)$. (7 marks)

IV. (a) (i) Using Cayley-Hamilton theorem find A^{-1} for $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$. (8 marks)

(ii) Find λ so that $\lambda\left(x^2 + y^2 + z^2\right) + 2xy - 2yz + 2zx$. (7 marks)

Or

(b) Reduce the quadratic form $3x^2 - 2y^2 - z^2 + 12yz + 8zx - 4xy$ into canonical form by orthogonal reduction. Also identify the nature of the quadratic form.

(15 marks)

V. (a) (i) Obtain the Fourier series expansion for the function $f(x) = x^2, -\pi < x < \pi$. Hence show that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$.

(8 marks)

(ii) Expand the function $f(x)$ defined by $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$ in Fourier series.

(7 marks)

Or

(b) (i) The following values of y gives the displacement (in cm) of a certain machine part for the rotation x of the flywheel. Express y in the form of a Fourier series upon first harmonic :

x :	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$
y :	0	9.2	14.4	17.8	17.3	11.7

(8 marks)

(ii) Find the half range sine series for $f(x) = \cos x,\ 0 < x < \pi$. (7 marks)

[4 × 15 = 60 marks]