C 14942 (Pages 3) Name.............................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2011

EN 2K 102—MATHEMATICS—II

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

1. (a) Solve $\left(\cos x + \tan y + \cos(x+y)\right) dx + \left(\sin x \sec^2 y + \cos(x+y)\right) dy = 0.$

(b) Solve $(D-2)^2 y = x^2$.

(c) Find the Laplace Transform of $t^2 e^t \sin 4t$.

(d) Find $L^{-1}\left(\dfrac{4s+5}{(s+1)^2 (s-2)}\right)$.

(e) Find the values of the constants λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda+2)x$ and $4x^2 y + z^3 = 4$ may intersect orgthogonally at the point $(1, -1, 2)$.

(f) Evaluate divergence and curl of the vector $(xy \sin z) i + \left(y^2 \sin x\right) j + \left(z^2 \sin xy\right) k$ at the point $\left(0, \dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

(g) Evaluate $\iint_R x^2 y^2 \, dxdy$ where R is the region in the first quadrant bounded by $x = 0, y = 0$ and $x^2 + y^2 = 1$.

(h) Show that the area bounded by a simple closed curve C is given by $\dfrac{1}{2}\int_C xdy - ydx$.

(8 × 5 = 40 marks)

2. (a) (i) Solve $x^2 \dfrac{d^2 y}{dx^2} + 2x\dfrac{dy}{dx} - 12y = x^3 \log x$. (8 marks)

(ii) Find the orthogonal trajectories of the family of curves $x^2 + y^2 + 2\lambda y = 0$, λ being the parameter.

(7 marks)

Or

Turn over

(b) (i) Solve $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$. (6 marks)

(ii) Solve $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$. (9 marks)

3 (a) (i) Find $L\left(\frac{e^{at} - \cos bt}{t}\right)$. (3 marks)

(ii) Find $L^{-1}\left(\log\left(\frac{s+1}{s}\right)\right)$. (3 marks)

(iii) Using Laplace Transform method, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t} \sin t$ given $y(0) = 0$, $y'(0) = 1$.

(9 marks)

Or

(b) (i) Prove that $\frac{\beta(m+1, n)}{\beta(m, n)} = \frac{m}{m+n}$. (5 marks)

(ii) Solve the simultaneous equations $\frac{dy}{dt} - wx = a \cos pt$; $\frac{dx}{dt} + wy = a \sin pt$. Given that $y(0), x(0) = 0$.

(10 marks)

4. (a) (i) Find the directional derivative of the function $\phi = 4xz^3 - 3x^2 yz^2$ at (2, – 1, 2) along the z-axis.

(4 marks)

(ii) For a solenoidal vector $\vec{F}$, prove that curl curl curl curl $\vec{F} = \nabla^4 \vec{F}$.

(6 marks)

(iii) If $\frac{d\vec{u}}{dt} = \vec{w} \times \vec{u}$ and $\frac{d\vec{v}}{dt} = \vec{w} \times \vec{v}$, prove that $\frac{d}{dt}\left(\vec{u} \times \vec{v}\right) = \vec{w} \times \left(\vec{u} \times \vec{v}\right)$.

(5 marks)

Or

(b) (i) Find the values of the constants *a, b, c* for which the vector $\vec{V} = (x + y + az)\, i + (bx + 3y - z)j + (3x + cy + z)\, k$ is irrotational. Find the corresponding scalar potential.

(8 marks)

(ii) If $\vec{r} = xi + yj + zk$, prove that $\nabla^2 \left(\frac{1}{r}\right) = 0$. (7 marks)

5. (a) (i) Evaluate $\iint_S \vec{A} \cdot \hat{n}\, dS$ where $\vec{A} = \left(x + y^2\right) i - 2xj + 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

(8 marks)

(ii) Change the order of integration and evaluate the integral $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}}\, dy\, dx$.

(7 marks)

Or

(b) (i) Apply Stoke's theorem to evaluate $\int_C (x + y)\, dx + (2x - 2)\, dy + (y + z)\, dz$ where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

(9 marks)

(ii) If $\vec{F} = (3x^2 + 6y)\, i - 14yzj + 20xz^2 k$, evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the path $x = t$, $y = t^2$ and $z = t^3$.

(6 marks)

[4 × 15 = 60 marks]