C 6283 (Pages : 2) Name.................................

Reg. No..............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, MAY 2010

PTEN/EN 09 101—ENGINEERING MATHEMATICS—I

(2009 admissions)

Time : Three Hours Maximum : 70 Marks

Part A

Answer **all** *questions.*

1. Give the formula for curvature of any given curve in Cartesian form.
2. What is D'Alembert's ratio test ?
3. State Cayley-Hamilton Theorem.
4. Find the eigenvalues of $2A^2$, if $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$.
5. Express $f(x) = x$ as a Fourier series in the interval $-\pi < x < \pi$.

(5 × 2 = 10 marks)

Part B

Answer any **four** *questions.*

6. Discuss the convergence of $\frac{5}{2} - \frac{7}{4} + \frac{9}{6} - \frac{11}{8} + \ldots$.
7. Find the centre of curvature of the parabola $y^2 = 12x$ at the point (3, 6).
8. Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.
9. Find the eigenvalues of adjacent matrix A, given

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$

10. Show that a constant "C" can be expanded in a infinite series $\frac{4c}{\pi}\left\{\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right\}$ in the range $0 < x < \pi$.
11. Develop $f(x)$ in Fourier series in the interval (–2, 2) if

$$f(x) = 0, \; -2 < x < 0$$
$$= 1, \quad 0 < x < 2.$$

(4 × 5 = 20 marks)

Turn over

Part C

Answer section (a) **or** *section (b) of each question.*
Each question carries 10 *marks.*

12. (a) Find the equivalent of

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} \text{ in polar co-ordinates.}$$

Or

(b) If $x = r\cos\theta$, $y = r\sin\theta$, verify that $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

13. (a) Test whether the series

$$1+\frac{1}{2^2}-\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}-\frac{1}{7^2}-\cdots \text{ is convergent or not ?}$$

Or

(b) State the values of x for which the following series converge

$$\frac{1}{1-x}+\frac{1}{2(1-x)^2}+\frac{1}{3(1-x)^3}+\ldots+\infty.$$

14. (a) Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ to canonical form by an orthogonal transformation.

Or

(b) Diagonalise the matrix A =

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

by means of an orthogonal transformation.

15. (a) Obtain Fourier series for the function $f(x)$ given by

$$f(x)=1+\frac{2x}{\pi}, = -\pi \le x \le 0$$

$$=1-\frac{2x}{\pi}, \quad 0 \le x \le \pi$$

Or

(b) Expand $f(x)=e^{-x}$ as a Fourier series in the interval $(-l, l)$.

(4 × 10 = 40 marks)