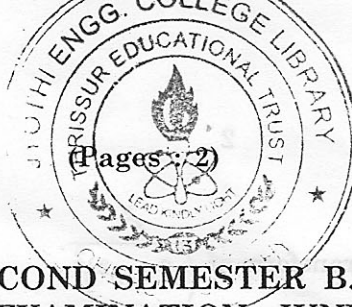


C 5801



Name.....

Reg. No.....

**COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)  
DEGREE EXAMINATION, JUNE 2010**

**EN 2K 102—MATHEMATICS—II**

(Common to all Branches)

Time : Three Hours

Maximum : 100 Marks

*Answer all the questions.*

1. (a) Solve  $(7x - 3y - 7) dx = (3x - 7y - 3) dy$ .
  - (b) Solve  $(D^2 + 1)y = x \cosh x$ .
  - (c) Show that  $\int_0^1 y^{p-1} \left( \log \frac{1}{y} \right)^{q-1} dy = \frac{\Gamma(q)}{p^q}$  where  $p > 0, q > 0$ .
  - (d) Find the Laplace transform of  $te^{3t} \sin 5t$ .
  - (e) If  $\vec{r}$  is the position vector of the point P  $(x, y, z)$ , prove that  $\nabla r^n = nr^{n-2} \vec{r}$  where  $r = |\vec{r}|$ .
  - (f) If  $\vec{F} = 3xyz^2 \mathbf{i} + 2xy^3 \mathbf{j} - x^2 yz \mathbf{k}$  and  $\phi = 3x^2 - yz$  find  $\mathbf{F} \cdot \nabla \phi$ .
  - (g) Evaluate by changing the order of integration  $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ .
  - (h) Evaluate  $\int \mathbf{F} \cdot d\mathbf{r}$  when  $\mathbf{F} = x^2 \mathbf{i} + y^2 \mathbf{j}$  along  $y = x^2$  in the  $xy$  plane from  $(0, 0)$  to  $(1, 1)$ .  
(8 × 5 = 40 marks)
2. (a) (i) Solve  $\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$ . (7 marks)
  - (ii) Solve Cauchy's equation  $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^2$ . (8 marks)
- Or
- (b) (i) Solve  $(D^2 + 2D + 5)y = e^x \sin 2x$ . (7 marks)
  - (ii) Solve  $(D^2 + 25)y = \tan 5x$  by the method of variation of parameters. (8 marks)
3. (a) (i) Using Gamma integral find the value of  $\int_0^{\infty} x^{7/2} e^{-x^2} dx$ . (7 marks)
  - (ii) Find  $L \left[ \frac{1 - \cos t}{t} \right]$  and  $L \left[ t^2 e^{2t} \cos 2t \right]$ . (8 marks)

Or

**Turn over**

(b) (i) Find the inverse Laplace transform of  $\frac{s^2}{(s^2 + w^2)^2}$ . (7 marks)

(ii) Find the Laplace transform of the periodic function :

$$f(t) = \begin{cases} t & 0 < t < a \\ 2a - t & a < t < 2a \end{cases} \text{ and } f(t + 2a) = f(t).$$

(8 marks)

4. (a) (i) Find the directional derivative of  $x^2 + y^2 + 4xyz$  at  $(1, -2, 2)$  in the direction of  $2i + 2j - k$ . (7 marks)

(ii) Show that  $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$  is irrotational. Find  $\phi$  such that  $\bar{F} = \nabla\phi$ . (8 marks)

Or

(b) (i) Find curl curl  $\bar{F}$  at  $(1, 2, -3)$  for  $\bar{F} = x^2i + y^2j + z^2k$ . (7 marks)

(ii) Prove that curl (grad  $\phi$ ) = 0 and div (curl  $\bar{F}$ ) = 0. (8 marks)

5. (a) (i) Evaluate  $\iiint_V xyz \, dx \, dy \, dz$  over the positive Octant of the sphere  $x^2 + y^2 + z^2 = a^2$ . (7 marks)

(ii) Verify Green's theorem in the plane for  $\int_C (3x^2 - 8y^2) \, dx + (4y - 6xy) \, dy$ , where C is the boundary of the region defined by  $x = 0$ ,  $y = 0$ ,  $x + y = 1$ . (8 marks)

Or

(b) (i) Verify divergence theorem for  $\bar{F} = 4xzi - y^2j + yzk$  over the cube bounded by  $x = 0$ ,  $x = 1$ ,  $y = 0$ ,  $y = 1$ ,  $z = 0$ ,  $z = 1$ . (7 marks)

(ii) Evaluate by Stoke's theorem  $\oint_C (\sin z \, dx - \cos x \, dy + \sin y \, dz)$  where C is the boundary of the rectangle  $0 \leq x \leq \pi$ ,  $0 \leq y \leq 1$  and  $z = 3$ . (8 marks)

[4 × 15 = 60 marks]