C 5801 (Pages : 2) Name.................................

Reg. No...............................

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2010

EN 2K 102—MATHEMATICS—II

(Common to all Branches)

Time : Three Hours Maximum : 100 Marks

Answer **all** *the questions.*

1. (a) Solve $(7x - 3y - 7)\, dx = (3x - 7y - 3)\, dy$.

   (b) Solve $(D^2 + 1)\, y = x \cosh x$.

   (c) Show that $\int_0^1 y^{p-1}\left(\log\frac{1}{y}\right)^{q-1} dy = \frac{\Gamma(q)}{p^q}$ where $p > 0$, $q > 0$.

   (d) Find the Laplace transform of $te^{3t} \sin 5t$.

   (e) If $\bar{r}$ is the position vector of the point P (x, y, z), prove that $\nabla r^n = nr^{n-2}\,\bar{r}$ where $r = |\bar{r}|$.

   (f) If $\overline{F} = 3xyz^2 i + 2xy^3 j - x^2 yzk$ and $\phi = 3x^2 - yz$ find $F \cdot \nabla\phi$.

   (g) Evaluate by changing the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$.

   (h) Evaluate $\int F \cdot dr$ when $F = x^2\, i + y^2 j$ along $y = x^2$ in the xy plane from (0, 0) to (1, 1).

(8 × 5 = 40 marks)

2. (a) (i) Solve $\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$. (7 marks)

   (ii) Solve Cauchy's equation $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = x^2$. (8 marks)

   Or

   (b) (i) Solve $(D^2 + 2D + 5)\, y = e^x \sin 2x$. (7 marks)

   (ii) Solve $(D^2 + 25)\, y = \tan 5x$ by the method of variation of parameters. (8 marks)

3. (a) (i) Using Gamma integral find the value of $\int_0^\infty x^{7/2} e^{-x^2}\, dx$. (7 marks)

   (ii) Find $L\left[\frac{1 - \cos t}{t}\right]$ and (8 marks)

   $L\left[t^2 e^{2t} \cos 2t\right]$.

   Or

Turn over

(b) (i) Find the inverse Laplace transform of $\frac{s^2}{\left(s^2+w^2\right)^2}$. (7 marks)

(ii) Find the Laplace transform of the periodic function :

$$f(t)=\begin{cases} t & 0<t<a \\ 2a-t & a<t<2a \end{cases} \text{ and } f(t+2a)=f(t).$$

(8 marks)

4. (a) (i) Find the directional derivative of x^2+y^2+4xyz at $(1,-2,2)$ in the direction of $2i+2j-k$. (7 marks)

(ii) Show that $\overline{F}=\left(6xy+z^3\right)i+\left(3x^2-z\right)j+\left(3xz^2-y\right)k$ is irrotational. Find ϕ such that $\overline{F}=\nabla\phi$.

(8 marks)

Or

(b) (i) Find curl curl $\overline{F}$ at $(1, 2, -3)$ for $\overline{F}=x^2i+y^2j+z^2k$. (7 marks)

(ii) Prove that curl (grad ϕ) = 0 and div (curl $\overline{F}$) = 0. (8 marks)

5. (a) (i) Evaluate $\iiint\limits_V xyz\,dx\,dy\,dz$ over the positive Octant of the sphere $x^2+y^2+z^2=a^2$.

(7 marks)

(ii) Verify Green's theorem in the plane for $\int\limits_C\left(3x^2-8y^2\right)dx+(4y-6xy)\,dy$, where C is the boundary of the region defined by $x=0\ y=0\ x+y=1$.

(8 marks)

Or

(b) (i) Verify divergence theorem for $\overline{F}=4xzi-y^2j+yzk$ over the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$.

(7 marks)

(ii) Evaluate by Stoke's theorem $\oint\limits_C(\sin z\,dx-\cos x\,dy+\sin y\,dz)$ where C is the boundary of the rectangle $0\leq x\leq\pi, 0,\leq y\leq 1$ and $z=3$.

(8 marks)

[4 × 15 = 60 marks]