



C 5800

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Name.....

Reg. No.....

COMBINED FIRST AND SECOND SEMESTER B.TECH. (ENGINEERING)  
DEGREE EXAMINATION, JUNE 2010

EN 2K 101—MATHEMATICS—I

(Common to all Branches)

Time : Three Hours

Maximum : 100 Marks

Answer all the questions.

1. (a) Evaluate  $\lim_{x \rightarrow 1} \frac{1-x}{\log x}$ .
- (b) Using Maclaurin's series, find the expansion of  $\sin x$ .
- (c) The radius of a sphere is found by measurement to be 18.5 inches with a possible error of 0.1 inch. Find the consequent error possible in the surface area as calculated from this measurement.
- (d) Find the  $n$ th derivative of  $x^2 e^{2x}$ .
- (e) Find the rank of the matrix  $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ .
- (f) Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ .
- (g) Find the Fourier series of  $f(x) = 1 - x^2$ ,  $-1 < x < 1$ .
- (h) Obtain the half-range sine series of  $f(x) = x$ ,  $0 < x < 2$ .

(8 × 5 = 40 marks)

2. (a) (i) Find the radius of curvature of  $\sqrt{x} + \sqrt{y} = 1$  at  $(1/4, 1/4)$ . (8 marks)

- (ii) If  $u = \tan^{-1} \left( \frac{x^3 + y^3}{x - y} \right)$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ . (7 marks)

Or

- (b) (i) Examine the function  $f(x, y) = x^3 + y^3 - 3x - 12y + 20$  for extreme values. (8 marks)

- (ii) Find the evolute of the parabola  $y^2 = 4ax$ . (7 marks)

3. (a) (i) Test for convergence :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$$

(8 marks)

- (ii) If  $y = \sin^{-1}x$ , prove that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ . (7 marks)

Or

Turn over

(b) (i) Test for convergence :

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

(8 marks)

(ii) Discuss the convergence of  $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ .

(7 marks)

4. (a) (i) Solve by using matrix inversion method :  $2x + y = 4$  and  $5x - 2y = 1$ .

(8 marks)

(ii) Find the eigen values of the matrix  $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ .

(7 marks)

Or

(b) (i) Solve the following system of equations :

$$\begin{cases} 4x + 2y + z + 3w = 0 \\ 6x + 3y + 4z + 7w = 0 \\ 2x + y + w = 0 \end{cases}$$

(8 marks)

(ii) Verify Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ .

(7 marks)

5. (a) (i) Expand  $f(x) = x^3$ , in  $-\pi < x < \pi$  in Fourier series.

(8 marks)

(ii) Find a half range cosine series for  $f(x) = x^2$  in  $0 \leq x \leq \pi$ .

(7 marks)

Or

(b) (i) Expand  $f(x) = x + \pi$ ,  $-\pi < x < \pi$  in Fourier series.

(8 marks)

(ii) Obtain the half range sine series for  $e^x$  in  $0 < x < 1$ .

(7 marks)

[4 × 15 = 60 marks]