C 6155 (Pages 3) Name..........

Reg. No..........

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION, JUNE 2010

CS 04 604—GRAPH THEORY AND COMBINATORICS

(2004 Admissions)

Time : Three Hours Maximum : 100 Marks

Answer **all** *questions.*

I. (a) Define union and intersection of two graphs with examples.

(b) Define Hamiltonian paths and circuits.

(c) List the properties of trees.

(d) What do you mean by colouring of a graph.

(e) A club has five members M_1, M_2, M_3, M_4 and M_5. They are going to form a finance committee but haven't decided how large it should be. They may pick everyone on it, or just M_3 or M_1 and M_2 etc. How many possibilities are there ?

(f) It is required to seat 5 men and 4 women in a row, so that the women occupy the even places. How many such arrangements are possible ?

(g) Obtain the particular solution of the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = 2^r$.

(h) Write the generating function of the numeric function :

$$a_r = \begin{cases} 0 & r \text{ is odd} \\ 2^{r+1} & r \text{ is even} \end{cases}.$$

(8 × 5 = 40 marks)

II. (a) (i) Briefly explain incidence and adjacency matrices. Draw the graph G corresponding to the adjacency matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

(8 marks)

Turn over

(ii) Prove that a simple graph with 'n' vertices and 'k' components can have at most $(n-k)(n-k+1)/2$ edges.

(7 marks)

Or

(b) (i) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

(8 marks)

(ii) Prove that any *two* simple connected graphs with 'n' vertices all of degree two, are isomorphic.

(7 marks)

III. (a) (i) Prove that a graph with 'n' vertices, $(n-1)$ edges and no circuits is connected.

(8 marks)

(ii) Define Binary search tree with suitable example.

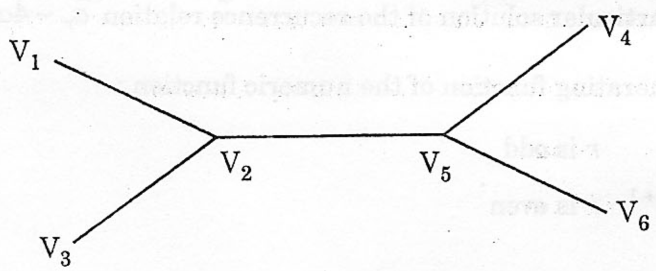
(7 marks)

Or

(b) (i) Prove that in any tree, there are atleast two pendant vertices.

(8 marks)

(ii) Find the radius and diameter of the tree given below :



(7 marks)

IV. (a) (i) Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible 2, 3 or 5.

(8 marks)

(ii) Out of a total of 130 students, 60 are wearing hats to class, 51 are wearing scarves and 30 are wearing both hats and scarves. Of the 54 students who are wearing sweaters, 26 are wearing hats, 21 are wearing scarves and 12 are wearing both hats and scarves. Every one wearing neither a hat nor a scarf is wearing groves.

1 How many students are wearing gloves ?

2 How many students not wearing a sweater are wearing hats but not scarves ?

(7 marks)

Or

(b) (i) In how many ways can two numbers be selected from integers 1, 2, 3, ..., 100 so that their sum is an even number ? An odd number ?

(8 marks)

(ii) It is known that 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog and 40% play bridge and jog. Someone claimed that 20% of the professors jog and play tennis and bridge. Would you believed the claim ? Why ?

(7 marks)

V. (a) (i) Determine the discrete numeric function corresponding to the following generating function

$$A(z) = \frac{7z^2}{(1-2z)(1+3z)}.$$

(8 marks)

(ii) Solve the recurrence relation :

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r.$$

(7 marks)

Or

(b) (i) Solve the recurrence relation $a_r - 5a_{r-1} + 6a_{r-2} - 2 = 2^r + r, r \geq 2$. Using generating function.

(8 marks)

(ii) Determine the discrete numeric function corresponding to the following generating function.

$$A(z) = \frac{z^5}{z - 6z + z^2}.$$

(7 marks)

[4 × 15 = 60 marks]