C 14750

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Name..............................

Reg. No..............................

SIXTH SEMESTER B.TECH. (ENGINEERING) DEGREE EXAMINATION DECEMBER 2010

CS 04 604—GRAPH THEORY AND COMBINATORICS

(2004 Admissions)

Time : Three Hours

Maximum : 100 Marks

Answer all *questions.*

I. (a) State and prove Euler's formula for planar graphs.

(b) Explain the Chinese postman problem.

(c) Define a tree. Explain its properties.

(d) Write the Kruskal's algorithm for finding minimal spanning tree.

(e) Explain the principle of inclusion and exclusion with example.

(f) How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 12$, with $x_i \geq 0$? How many solutions with $x_i \geq 1$?

(g) Solve $a_{n+2} - 2a_{n+1} + 4a_n = 0$

(h) Find the generating function of the sequence 0, 2, 6, 12, 20, 30, 42, ...

(8 × 5 = 40 marks)

II. (a) Show that for a connected graph, following are equivalent :

(i) G is Eulerian.

(ii) Degree f each vertex is even.

(iii) The edge set of the graph is partitioned into cycles.

(15 marks)

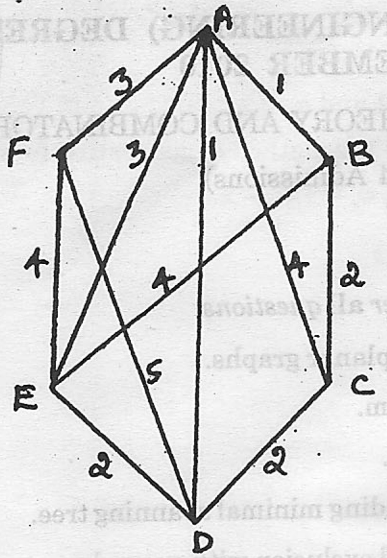
Or

(b) Prove that a graph G with n vertices has a Hamilton path if the sum of the degrees of every pair of vertices v_i, v_j, in G satisfying the condition $d(v_j) + d(v_j) \geq n-1$.

(15 marks)

Turn over

III. (a) Find the maximal anbd minimal spanning trees of the following graph :



(15 marks)

Or

(b) Show that any sub graph g of a connected graph G is contained in some spanning tree of G if and only if g contains no circuit.

IV. (a) (i) How many integral solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 22$, where $x_i \geq 0 \ \forall \ 1 \leq i \leq 4$ and $x_s \geq 3$.

(7 marks)

(ii) Show that if n and k are positive integers with $n = 3k$, then $\frac{n!}{(3!)^k}$ is an integer.

(8 marks)

Or

(b) (i) Determine the co-efficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$. (8 marks)

(ii) Determine the number of positive integers x where $x \leq 99, 99, 999$ and the sum of the digits in x equals 31.

(7 marks)

V. (a) (i) Find solution to Fibonacci relation $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. (7 marks)

(ii) Find the complete solution of $a_{n+1}\ 2a_{n-1} = n + 3$. (8 marks)

Or

(b) (i) Solve $a_n - a_{n-1} = 2\,(n-1)$ for $n \geq 1$ and $a_o = 2$. (7 marks)

(ii) Solve the recurrence relation by the method of generating function.

$$a_{n+2} - 2\,a_{n+1} + a_n = 2^n,\ n \geq 0,\ a_o = 1, a_1 = 2.$$

(8 marks)

[4 × 15 = 60 marks]