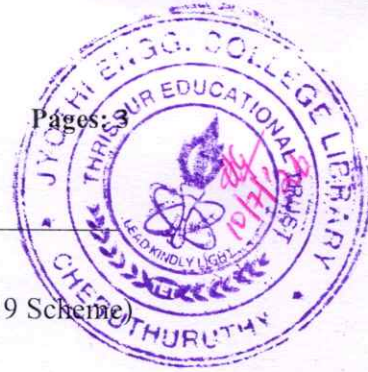


Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S4 (S, FE) (FT/WP/PT/S2PT) Examinations April/May 2026 (2019 Scheme)



Course Code: MAT204

Course Name: PROBABILITY, RANDOM PROCESSES AND NUMERICAL METHODS

Max. Marks: 100

Duration: 3 Hours

## PART A

(Answer all questions; each question carries 3 marks)

Marks

- 1 The pdf of a discrete random variable X is given by 3
- |      |   |                 |                |                 |
|------|---|-----------------|----------------|-----------------|
| x    | 0 | 1               | 2              | 3               |
| f(x) | k | 2k <sup>2</sup> | k <sup>2</sup> | 3k <sup>2</sup> |
- Find (i) k (ii) mean of X
- 2 Derive the mean and variance of Poisson distribution with parameter  $\lambda$  3
- 3 If X is normally distributed with mean 8 and standard deviation 4, find  $P[5 \leq X \leq 10]$  3
- 4 If the time in hours, required to repair a machine is exponentially distributed with mean 4, find the probability that the repair time exceeds 4 hours 3
- 5 Show that the power spectral density of a WSS process an even function 3
- 6 A random process is defined by  $X(t) = A \cos \omega t, t \geq 0$ , where  $\omega$  is a constant and A is uniformly distributed in (0,3). Find the mean of X(t) 3
- 7 Use Newton-Raphson method to evaluate the fourth root of 32 3
- 8 Evaluate  $\int_0^{\pi/2} \cos x \, dx$  using Trapezoidal rule in 6 subintervals 3
- 9 Use Euler's method to compute  $y(0.1)$ , where  $y' = x + xy + y, y(0) = 1$ . Take  $h=0.05$  3
- 10 Write the normal equations for fit a parabola of the form  $y=a+bx+cx^2$  to a given data by the method of least squares 3

## PART B

(Answer one full question from each module, each question carries 14 marks)

## Module -1

- 11 a) Derive the mean and variance of a binomially distributed random variable 7
- b) It is known that 2% of the bolts produced by a company are defective. The bolts are supplied in boxes of 200 bolts. What is the probability that a randomly 7

chosen box contained more than 5 defective bolts? In a consignment of 1000 such boxes how many can be expected to have more than 5 defective bolts?

- 12 a) There are 250 typographical errors in a book of 1000 pages. The number of errors per page is supposed to follow Poisson distribution. What is the probability that a randomly selected page will have more than 2 errors? 7
- b) The joint PDF of two continuous random variables is given by 7
- $$f(x, y) = kxy, 0 < x < 4, 1 < y < 5.$$
- Find (i) k (ii) marginal distributions of X and Y.

### Module -2

- 13 a) The marks obtained by a batch of students in a certain subject are normally distributed. 10% of the students got less than 45 marks and 5% got more than 75. Find the mean and variance of the distribution 7
- b) Find the mean and variance of the random variable with probability density function  $f(x) = k(10 - x)x^2, 0 \leq x \leq 1$  7
- 14 a) A string of length 1 meter is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twice the length of the other? 7
- b) Two random variables X and Y have the joint probability distribution function given by 7
- $$f(x, y) = \begin{cases} e^{-(x+y)}, & x \geq 0, y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$
- Find (a) the marginal densities of X and Y (b) Check whether X and Y are independent.

### Module -3

- 15 a) Show that the random process defined by  $X(t) = A \cos(\omega t + \theta)$  is a WSS process where A and  $\omega$  are constants and  $\theta$  is a uniform variable in  $(0, 2\pi)$  7
- b) The autocorrelation of the random telegraph signal process is given by 7
- $$R(\tau) = a^2 e^{-2\alpha|\tau|}.$$
- Determine the power spectral density of the random telegraph signal
- 16 a) Cell-phone calls processed by a certain wireless base station arrive according to a Poisson process with an average of 12 per minute. (i) What is the probability that more than three calls arrive in an interval of length 20 seconds? (ii) What is the probability that more than 2 calls arrive in each of two consecutive intervals 7

of length 30 seconds?

- b) A fair coin is tossed and based on the outcome a random process  $X(t)$  is defined as 7

$$\text{follows: } X(t) = \begin{cases} \sin 2t, & \text{if head appears} \\ t, & \text{if tail appears} \end{cases}, \text{ where } -\infty < t < \infty$$

Find the mean and variance of the process

#### Module -4

- 17 a) Solve the equation  $3x + \sin x - e^x = 0$  by Regula-Falsi method 7
- b) Find the interpolating polynomial, the graph of which passes through the points  $(-1, 3), (0, -4), (1, 5), (2, -6)$  using Lagrange's interpolation 7
- 18 a) The following table gives the values of  $\sin \theta$  where  $\theta$  is in degrees. Using Newton's forward interpolation formula estimate the value of  $\sin(8^\circ)$  7

| $\theta$      | 5      | 10     | 15     | 20    | 25     | 30  |
|---------------|--------|--------|--------|-------|--------|-----|
| $\sin \theta$ | 0.0871 | 0.1736 | 0.2588 | 0.342 | 0.4226 | 0.5 |

- b) Compute  $\int_0^1 \frac{1}{1+x^2} dx$  using Simpson's one third rule with step-size  $h = 0.25$  7

#### Module -5

- 19 a) Use Gauss-Seidel method to solve the system:  $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$  7
- b) Using modified Euler method (Runge-Kutta method of order II), find the value of  $y$  at  $x = 0.25$  and  $x = 0.5$ , given  $\frac{dy}{dx} = 2xy$ ,  $y(0) = 1$  7
- 20 a) Use method of least squares to find the unique quadratic polynomial, the graph of which passes through the data points  $(0, 0), (2, 2), (3, 6), (4, 12)$  7
- b) Using fourth order Runge-Kutta method, evaluate  $y(0.4)$ , given by  $\frac{dy}{dx} = x^2 + y^2$ ,  $y(0) = 1$ . Use  $h = 0.2$  7

\*\*\*