Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1 (S,FE) S2 (S,FE) Examinations May 2026 (2019 Scheme)

Course Code: MAT102

Course Name: VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND TRANSFORMS

(2019 SCHEME)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all Questions. Each question carries 3 Marks Marks

1 Find curl (curl $\bar{F}$) where $\bar{F} = x^2y\hat{\imath} - 2xz\,\hat{\jmath} + 2yz\hat{k}$ (3)

2 Find the work done by the force $\bar{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$ when it moves a particle along the curve $y = 2x^2$ in the xy-plane from (0,0) to (1,2). (3)

3 Use Green's theorem evaluate the line integral $\int_c (x - 2y)dx + xdy$ taken once around the circle $x^2 + y^2 = a^2$ and positively oriented. (3)

4 Determine the flux of the vector field $\bar{F} = x\hat{\imath} + y\hat{\jmath}$ across the outward oriented sphere $x^2 + y^2 + z^2 = a^2$. (3)

5 Show that the functions x and $xln(x)$ are linearly independent. Hence form an ODE for the given basis $\{x, xln(x)\}$ (3)

6 Solve the initial value problem $y'' - 2y' + 5y = 0, y(0) = -3, y'(0) = 1$. (3)

7 Find the Laplace Transform of $e^{-4t}sin3t$. (3)

8 Find the inverse Laplace Transform of $\frac{e^{-s}}{(s+4)^3}$ (3)

9 Find the Fourier sine transform of $f(x) = 3x, 0 < x < 6$. (3)

10 Given the Fourier Transform of $f(x) = e^{-x^2}$ is $\frac{1}{\sqrt{2}}e^{\frac{-\omega^2}{4}}$, find the Fourier Transform of xe^{-x^2}. (3)

PART B

Answer one full question from each module, each question carries 14 marks

Module I

11 (a) If $\bar{A} = (3x^2+6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$, evaluate $\int \vec{A}\,.\,\overrightarrow{dr}$ from (0,0,0) to (1,1,1) along the path $x = t, y = t^2, z = t^3$. (7)

(b) Show that $\bar{F} = (y^2\cos x + z^3)\hat{\imath} + (2y\sin x - 4)\hat{\jmath} + 3xz^2\hat{k}$ is conservative and hence find its scalar potential (7)

OR

12 (a) Find the values of the constants a,b,c so that $\bar{\bar{F}}$= (axy+bz^3)$\hat{\imath}$ + (3x^2- cz)$\hat{\jmath}$ + (3xz^2- y)$\hat{k}$ may be conservative. For these values of a,b, c find the scalar potential of $\bar{F}$. (7)

(b) Show that $\int_c (3x^2 e^y dx + x^3 e^y dy)$ is independent of the path and hence evaluate the integral from (0.0) to (3,2) (7)

Module II

13 (a) Use divergence theorem to evaluate $\iint_s \bar{F}.\hat{n}ds$ where $\bar{F} = 5\hat{\imath} + 10y\hat{\jmath} + z^3\hat{k}$ and S is the surface $-1 \leq x \leq 1, \frac{1}{2}x \leq y \leq x, 0 \leq z \leq y$. (7)

(b) Apply Stoke's theorem to evaluate $\int_c (ydx + zdy + xdz)$ where C is the curve of intersection of $x^2+y^2+z^2 = a^2$ and x +z =a such that its boundary curve with counter clockwise orientation looking down the positive Z-axis. (7)

OR

14 (a) Use Stoke's theorem to evaluate $\int_c \bar{F}.dr$ where $\bar{F}$ = (x-y) $\hat{\imath}$ + (y-z)$\hat{\jmath}$+ (z –x)$\hat{k}$ and C is the boundary of the portion of the plane x + y+ z = 1 in the first octant. (7)

(b) Apply Green's Theorem to evaluate $\int_c (y - \sin x)dx + \cos x\, dy$ where C is the plane triangle enclosed by the lines y = 0, x = $\frac{\pi}{2}$ and y = $\frac{2x}{\pi}$. (7)

Module III

15 (a) Solve the initial value problem $x^2y'' + 3xy' + 0.75y = 0$ given that y(1) = 1, $y'(1)$ = -2.5. (7)

(b) Solve by the method of undetermined coefficients, $y'' - 4y = xe^x + cos\, 2x$. (7)

OR

16 (a) Solve the differential equation $y'' - 4y' + 13y = e^{2x}cos\, x$ using method of undetermined coefficients. (7)

(b) Solve $(D^2 + 2D + 1)y = 4e^{-x}logx$ using the method of variation of parameters (7)

Module IV

17 (a) Find the inverse Laplace transform of (7)

(1) F(s) = $\frac{2(e^{-s} - e^{-3s})}{s^2-4}$ (2) $\frac{1}{s^3-a^3}$

(b)

Find the Laplace transform of $\frac{1-cost}{t^2}$. (7)

OR

18 (a) Using Laplace Transform solve $y'' - 3y' + 2y = 4t + e^{3t}$ given $y(0) = 1$, $y'(0) = -1$. (7)

(b) Using Convolution theorem, find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)^2}$. (7)

Module V

19 (a) Find the Fourier Transform of f(x) = $e^{-\alpha|x|}$, $\alpha > 0$. Using this result find the inverse transform of $\frac{1}{1+\omega^2}$ and hence prove that $\int_0^\infty \frac{cos\omega x}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-|x|}$. (7)

(b) Using Fourier Integral show that $\int_0^\infty \frac{sin\omega cos\omega x}{\omega} d\omega = \frac{\pi}{2}$ 0<x<1. (7)

OR

20 (a) Find the Fourier sine and cosine transform of f(x) = $\begin{cases} 1: & if\ 0 \le x \le c \\ 0: & if\ x > c \end{cases}$. (7)

(b) Find the Fourier Transform and Fourier integral representation of the function f(x) = $\begin{cases} 1-|x|: & for\ |x| < 1 \\ 0 & : otherwise \end{cases}$. Hence prove that $\int_0^\infty \frac{1-cosx}{x^2} dx = \frac{\pi}{2}$. (7)

****