Reg No.:_____ Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S1 (S,FE) S2 (S,FE) Examinations May 2026 (2019 Scheme)

Course Code: MAT101

Course Name: LINEAR ALGEBRA AND CALCULUS

(2019 -Scheme)

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 3 marks Marks

1 (3)

Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 1 \\ 0 & 4 & 8 \\ 2 & 0 & 4 \end{bmatrix}$

2 Are the following set of vectors linearly independent? Justify your answer. (3)

$(3 \quad -1 \quad 4), (6 \quad 7 \quad 5), (9 \quad 6 \quad 9)$

3 Given $f(x, y) = e^x siny$, show that the function satisfies the equation (3)

$f_{xx} + f_{yy} = 0$

4 Find the slope of the surface $z = \sqrt{3x + 2y}$ in the Y-direction at the point $(2, 5)$ (3)

5 Evaluate $\int_1^a \int_1^b \frac{1}{xy} dy\, dx$ (3)

6 Find the area of the region R enclosed by $y = 1, y = 2, x = 0, x = y$. (3)

7 Determine whether the following sequence converges, and if so, find its limit (3)

$1, \frac{1}{2}, \frac{1}{2^2}, \frac{1}{2^3}, \ldots$

8 Determine whether the given series converges,and if so, find its sum. $\sum_{k=1}^{\infty} 3^{2k}5^{1-k}$ (3)

9 Obtain Fourier sine series for $f(x) = x$ in the interval $0 \leq x \leq \pi$ (3)

10 Find the Maclaurin's series for $f(x) = \frac{1}{1-x}$ (3)

PART B

Answer one full question from each module, each question carries 14 marks.

MODULE 1

11 (a) Solve the given linear system, using Gauss elimination method. (7)

$y + z - 2w = 0, 2x - 3y - 3z + 6w = 2, 4x + y + z - 2w = 4$

(b) Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ (7)

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(a) Diagonalize the matrix $A = \begin{bmatrix} 6 & 0 & 0 \\ 12 & 2 & 0 \\ 21 & -6 & 9 \end{bmatrix}$ (8)

(b) Solve the following linear system using Gauss elimination method. $x_1 - x_2 + x_3 = 0,\ -x_1 + x_2 - x_3 = 0, 10x_2 + 25x_3 = 90,\ 20x_1 + 10x_2 = 80$ (6)

MODULE 2

13 (a) If $z = {}^{x}/_{y}$, where $x = 2cosu\,, y = 5sinv$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ (7)

(b) Locate all relative extrema and saddle points of

$f(x,y) = 3x^2 - 2xy + y^2 - 8y.$ (7)

14 (a) Find the local linear approximation L of $f(x, y, z) = \ln(x + yz)$ at the point P(2,1,-1).Compute the error in approximation f by L at the point Q(2.02,0.97, -1.02). (7)

(b) If $w = 4x^2 + 4y^2 + z^2$, $x = \rho sin\varphi cos\theta$, $y = \rho sin\varphi sin\theta$, $z = \rho cos\varphi$ $find$ $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$ using chain rule. (7)

MODULE 3

15 (a) Evaluate the integral by reversing the order of integration $\int_0^2 \int_{\frac{y}{2}}^1 cos(x^2)\, dx\, dy$ (7)

(b) Evaluate $\iint_R xy\, dA$, where R is the region enclosed by $y = \sqrt{x}$, $y = 6 - x$ and $y = 0$ (7)

16 (a) Use double integral to find the volume of the tetrahedron bounded by the coordinate planes and the plane $z = 4 - 4x - 2y$ (7)

(b) Evaluate the integral converting to polar coordinates $\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{\sqrt{1+x^2+y^2}}\, dx\, dy$ (7)

MODULE 4

17 (a) (i) Determine whether the series converges. $1 + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{3}} + \cdots + \frac{1}{\sqrt[3]{k}}$.. (7)

(ii)Use limit comparison test to determine whether the following series converges or diverges $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}+1}$

(b) Use ratio test for absolute convergence to determine whether the series converges $\sum_{k=1}^{\infty} (-1)^k \frac{(2k-1)!}{3^k}$ (7)

18 (a) Prove that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ is convergent. (7)

(b) Determine whether the given series converges absolutely or

Converges Conditionally $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k+3}{k(k+1)}$ (7)

MODULE 5

19 (a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$ (7)

(b) Find the Fourier series expansion for $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ (7)

20 (a) Find the Fourier series expansion for $f(x) = 2x - x^2$ in $(0, 3)$ (7)

(b) Obtain half range cosine series for $f(x) = \begin{cases} kx, & 0 \le x \le \frac{l}{2} \\ k(l-x), & \frac{l}{2} \le x \le l \end{cases}$ (7)

Deduce the sum of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

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