Reg No.:_______________ Name:_______________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

B.Tech Degree S3 (S, FE) (FT/WP/S1PT) Examinations May/June 2026 (2019 Scheme)

Course Code: MAT201

Course Name: Partial Differential equations and Complex analysis

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions. Each question carries 3 marks Marks

1	Derive a partial differential equation from the equation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	(3)
2	Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 y$	(3)
3	Write down the possible solutions of the one-dimensional heat equation	(3)
4	Find the steady state temperature distribution in a rod of length $25\ cm$, if the ends of the rod are kept at $10^0 C$ and $85^0 C$ respectively.	(3)
5	Show that an analytic function with constant real part is constant.	(3)
6	Find the fixed points of $w = \frac{2iz-1}{z+2i}$	(3)
7	Evaluate $\oint_C \sec z\, dz$, where C is the unit circle counter clockwise	(3)
8	Evaluate $\oint_C \frac{\sin z}{z-4} dz$, $C : \lvert z - 4 - 2i \rvert = 6.5$	(3)
9	Determine and classify the singularity of $f(z) = z^2 \sin(\frac{1}{z})$	(3)
10	Find the singular point and residue of the function $f(z) = \frac{1-\cos z}{z^3}$	(3)

PART B

Answer any one full question from each module. Each question carries 14 marks

Module 1

11	(a)	Form the partial differential equation by eliminating the arbitrary function from $f(xy + z^2,\ x + y + z) = 0$.	(7)
	(b)	Solve $x(y - z)p + y(z - x)q = z(x - y)$	(7)
12	(a)	Solve $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 2u$, using method of separation of variables.	(7)
	(b)	Solve $(p^2 + q^2)x = pz$ using Charpit's method.	(7)

Module 2

13 (a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = 3\sin(\pi x/l)$ from which it is released at time $t = 0$. Find the displacement $y(x,t)$ at any time t. (7)

(b) An insulated rod of length 10 cm has its ends A and B maintained at 0^0C and 90^0C respectively until steady state conditions prevail. If B is suddenly reduced to 0^0C and maintained at 0^0C, find the temperature function $u(x,t)$. (7)

14 (a) A string of length 20 cm fastened at both ends is displaced from its position of equilibrium, by imparting to each of its points an initial velocity given by $v = \begin{cases} x, & 0 \le x \le 10 \\ 20 - x, & 10 \le x \le 20 \end{cases}$, x being the distance from one end. Determine the displacement at any subsequent time. (7)

(b) Find the temperature $u(x,t)$ in a bar whose ends are kept at 0^0C and whose initial temperature is $f(x) = x(10 - x)$ given that its length is 10 cm. (7)

Module 3

15 (a) Test the continuity at $z = 0$, if $f(z) = \begin{cases} \frac{(Im\, z^2)}{|z|^2}, & z \ne 0 \\ 0, & z = 0 \end{cases}$ (7)

(b) Show that $v = 3x^2y - y^3 + x^2 - y^2$ is harmonic. Also find the harmonic conjugate of v. (7)

16 (a) Prove that the function $f(z) = \cos z$ is analytic and find its derivative (7)

(b) Find the images of the lines $x = 2$ and $y = 3$ under the transformation $w = z^2$ (7)

Module 4

17 (a) Evaluate $\int_0^{2+i} Re(z)\, dz$ along the line segment from 0 to $2 + i$. (7)

(b) Evaluate $\oint_C \frac{e^z}{(z-2i)^2} dz$, C is the circle $|z - 3i| = 2$ using Cauchy's integral formula. (7)

18 (a) Evaluate $\oint_C \frac{e^{2z}}{(z-1)(z-2)} dz$, C is the circle $|z| = 3$ using Cauchy's integral formula. (7)

(b) Find the Taylor series of $f(z) = \frac{1}{z^2}$ about $z = -i$ (7)

Module 5

19 (a) Find the Laurent's series of $f(z) = \frac{1}{z(z-1)}$ in $1 < |z+1| < 2$. (7)

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ (7)

20 (a) Evaluate $\oint_C \frac{z^2}{(z-1)^2(z+2)} dz$, where $C: |z+2| = 4$ using Cauchy's residue theorem. (7)

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2+1}{(x^2+4)(x^2+9)} dx$ (7)

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